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ABSTRACT

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The Correlation Coefficient as the Ratio of Two Means: An Interpretation Due to Galton and Brogden

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Abstract

A standard interpretion of the coefficient of correlation may be obtained from the ratio of two variances. If it is assumed that two variables, X and Y, are bivariate normal and scaled to have zero means and unit variances, it can be shown that $\rho(XY)$ is the ratio of two conditional means. Galton's statistical studies of inheritance used the "ratio of means" expression for the correlation coefficient to illustrate the proportion of regression towards the mean for the progeny of parents in the extremes of a distribution for a selected trait. In the area of personnel selection, Brogden showed that $\rho(XY)$ may be interpreted as the ratio of the mean criterion score for persons selected on a test to the mean criterion score for "perfect" selection (i.e., selecting on the basis of criterion score). Monte Carlo studies indicate that the Galton--Brogden approach may be useful for dealing with extreme group studies.

The Correlation Coefficient as the Ratio of Two Means: An Interpretation Due to Galton and Brogden

Depending upon the particular event one picks as the birth of the correlation coefficient, we are only a few years away from its centennial, one way or the other. This is no true anniversary; until the last quarter of the nineteenth century relationships between variables were generally expressed in terms of deterministic formulas based on cause-and-effect considerations. Although least squares data fitting had been in use since the time of Legendre and Gauss, its use was generally limited to cases in which data were sparse or especially prone to measurement error. Even in such instances, the fit of least squares descriptions was often expressed in terms of the raw error of estimation for a given known data point (some of Einstein's work gives examples of this practice). It was not until Francis Galton turned his attention to heredity that an index was developed to express the strength of relationship between two variables in which the fit to all the data was considered.

We are accustomed to thinking of the correlation coefficient as the ratio of the covariance of two measures to the product of their standard deviations (following Bravais and Pearson) or as the square root of the ratio of estimated score variance to observed score variance for either measure or as the geometric mean of the regression weights for estimating each variable from the other. We may employ one or another of these representations, depending upon the particular situation in which we find ourselves. An alternative view is the one which was originally offered by Galton and later used by Brogden, viz., the ratio of two conditional means. After briefly describing the development of this approach, we will consider a situation in which such an alternative representation might be especially useful.

In 1877 Galton presented a paper to the Royal Institute of Great Britain in which he postulated a mechanism he called 'reversion' as the force which maintained the stability of distributions of organismic characteristics across generations. He based this idea upon the observation of several generations of sweet peas, noting that on any given characteristic, the deviation of offspring from the average of all offspring was generally less than that of their parents from the average of all parents. Examples of this type are commonly used in those texts that choose to even mention Galton.

Over the next eight years Galton collected data on successive human generations in order to test the generality of the reversion hypothesis. (In order to get an acceptable number of cases, Galton offered substantial rewards for family records in some instances.) He also looked for a numerical method of describing the strength of the reversion mechanism. In 1885 he presented a paper to the Royal Anthropological Institute in which he supported his original idea and offered the first version of the numerical method. Dealing with the now-famous data on the heights of parents and their adult children, Galton computed the mean height of each conditional distribution of children whose parents were of a given height themselves. He reported the average ratio of the childrens' heights (in deviations around the median height of all the children) to their parents' height (also in median-deviation form) as the index of reversion from parents to children. In so doing he made no correction for differences in the amount of variability in the two distributions, although he did note that the probable errors were different. He also reported that the index of the relationship from children to parents had to be computed separately and was not simply the reciprocal of the index for parent-to-child reversion.

Three years later came a new application and a revision in the method. His 1888 paper to the Royal Society (1) dealt with comparing two characteristics in

the same population rather than one characteristic across generations, and (2) described a computational method through which the reversion or correlation index had the same value regardless of the direction in which the effect was studied. The new version of his method consisted of 'transmuting' the deviations previously used to have equal statistical variability, i.e., he started using standard scores. Equation 1 on page 2 of the handout describes this situation for bivariate normal data. In practice this coefficient was computed for each conditional distribution and then averaged across distributions. W.F.R. Weldon made great use of both the ideas in the 1888 paper in his studies of marine life, and his 1892 paper on correlations among organs in shrimp gives a clearer explanation of the method than does Galton's writing. (It also represented one of the first uses of Galton's method with infrahuman zoological types).*

Equation 2 represents a reasonable alternative to computing a number of ratios and then averaging, viz., compute the mean of one variable over a range of values for the other variable, and divide by the mean of the conditionalizing variable over the same range. This is essentially the form used by Brogden (1946) as an expression for the correlation between a measure used as a selection instrument and a measure of criterion performance. If the X variable represents a test used for selection from a pool of applicants and Y is a criterion measure of job performance, then the numerator of this expression represents the mean level of on-the-job performance for those applicants selected by the test. If we assume a given proportion of applicants must be selected, whether by testing or by job performance, and that the distributions of test scores and performance scores have the same shape, then the denominator of this formula can be rewritten as the mean level of performance for those applicants selected on the basis of performance.

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*In 1896, Pearson demonstrated the superior properties of Bravais' product-moment coefficient, and the Galton-Weldon method began to fade from the literature.

A sample version of this is given in equation 3. The correlation coefficient thus represents the efficiency of the selection measure with respect to the performance criterion.

We had originally planned to present an expanded version of the foregoing. It is a very interesting story, especially when one reads the original sources, and we have had to omit some of the more delightful details. We would have finished with results from a Monte Carlo study examining the bias and variability in Galton's method relative to Pearson's. After we had completed the simulation, however, an idea occurred to us that derailed our original train of thought.

We first examined the estimator in equation 6 on page 2, called 'Galton z.' It is simply the average of two Galton-Brogden estimates, computed for equal-sized portions in the extremes of the joint distribution of scores in the sample. The results are shown in Table 1 of the handout. As stated, all empirical results are based on 1000 samples of 80 pairs of scores from bivariate normal distributions with correlations shown in the column heads. (We used bivariate normal data to circumvent the interpretational problems pointed out by John Carroll twenty years ago, to maintain the spirit of the data with which Galton originally worked, and to be able to compare the results with analytical predictions.)

The first line of Table 1, marked "Pearson" shows the mean and variance for 1000 replications of the Pearson product-moment coefficient with data for which the true correlation is 0.0, 0.2, 0.5, or 0.8. These results compare very favorably with the values given in the next line, which represent the mean coeefficient value computed from the formula given in Kendall and Stuart (1973), and the variance of the coefficients as computed from Pearson's (1896) formula. The slight negative bias in the simulated results should be borne in mind when examining the rest of the results.

The next major section of Table 1 shows simulated results for the method we called 'Galton z', as previously described. Here and throughout the tables, the proportions listed under the name of a technique refer to the proportion of the original sample of 80 scores that was taken from each tail of the distribution of the X variable for use in the actual computations. Examining the results for Galton z, we find virtually no bias in estimation, especially when the bias of the simulation program (mentioned earlier) is taken into account. The other main point of interest in these results concerns the variance of the estimates. Although the variance of the Galton z estimates is always greater than that of the Pearson method, there is a quadratic relationship between the proportion of scores used in the computation and the variance of the distribution of estimates. In each of the cases examined, the minimum variance occurs when approximately 30% is used from the upper and lower ends of the sample. Note that this means that estimates based upon the upper and lower 50% of the sample, using all the available data, are less efficient than those using only the upper and lower 20%.

At this point two questions occurred to us. First, how is it possible for a statistic to become more efficient when computed on less of the data? (Note: the same generated scores were used for the computation of all estimates. Thus the data used for the .1 version of Galton z are a subset of the data used for the .5 version, and all other estimates were computed using the same samples.) The second question was, are we not violating the old admonition of not computing correlation coefficients from extreme group data?

We have no answer to the first question. Our answer to the second question is, the admonition against extreme groups is always given in the context of computing Pearsonian coefficients, and it may not apply to Galton's method or its derivatives. Galton's method may be very useful in a situation in which one had extreme group data only and wished to assess the strength of relationship between

two variables. Furthermore if one variable is well known to the researcher, perhaps tabled for a population like census data, one might choose to randomly sample subjects from the extremes of that variable, observe them on various measures of interest, and apply Galton's method to determine the correlation between each of those variables and the variable on which they were selected.

In order to further examine the possible utility of this approach, we first decided to examine the effect of extreme group data on the Pearson coefficient. Using the same data as before, we computed the Pearson correlation coefficient for each proportion that had been used with Galton z. The results are given in Table 1 in the section marked 'Pearson Extreme.' For each population correlation other than 0.0, there is a marked positive bias in the mean estimate, as expected. For all population correlations other than 0.8, the variances of the estimates are considerably larger than those of the Pearson method when computed for the complete sample of scores, the same value given for the proportion of .5. From the standpoint of bias, the Galton z method would seem preferable for extreme group work.

An alternative approach that would effect the same savings in number of subjects to be sampled and observed is to sample the same number of observations that the Galton z method would use, but without restrictions as to their location in the joint distribution. The correlation between variables could be assessed by means of the Pearson r. The results of this approach are shown in the section labeled 'Pearson Same n.' The values given here were computed from the same formulas for mean and variance that were used with the 'Pearson' results and were not simulated. Again the bias is quite small on the average. The variances are uniformly larger than those of the original 'Pearson' method, due to the smaller sample sizes involved, but are also generally larger than those for Galton z. Generally speaking, the Galton z estimates are more efficient for proportions less than approximately .4.

Summarizing to this point, Galton's method (our version of it), when used with data selected from the extremes of the marginal distribution of one variable, 1) lacks the bias of Pearson's method when computed on the same data, and 2) is generally more efficient than Pearson's method when computed from the same number of freely chosen scores. There is, however, a drawback. In the simulation we followed Galton's procedure of computing the mean and variance for each variable on the basis of all scores, not just those that are ultimately used in the computations. To take advantage of the Galton z results, therefore, one would have to actually collect 80 pairs of scores, compute the mean and variance of each variable, rank the scores on one variable, and then select the portion that would be retained for computation. In such a setting, the decision as to whether to use Galton's or Pearson's method would hinge on the difference in the work required to rank and select subjects on one variable versus computing the sum of cross products for the full sample (and the increased efficiency of the Pearson r for the full sample).

We investigated three alternative approaches to Galton's method in an effort to find a reasonable technique for use in the sample. That is, we wanted to keep the lack of bias for extreme group situations without having to use information that could come only from a larger, inclusive sample. Our choices for these modifications may not have been optimal, but they were straightforward and easy to implement in the simulation program.

The first of these modifications was based on a method suggested by Brown (Mosteller, 1946), which essentially says that if the ratio of population standard deviations is known, then don't estimate the standard deviations in the sample, but use that known ratio. This might be unrealistic in practice, but it was easy enough to do for the simulation. Since the standard deviations for the simulated data were both unity, we simply eliminated the computation of sample standard deviations and used raw deviations from the mean of the retained scores in our computations.

The results of this simulation are shown in Table 2 in the section headed Galton Raw. Note that the means and variances are comparable to those of the original Galton z, perhaps a bit larger in general. To some extent this increase can be attributed to the fact that for correlations of 0.5 and 0.8, this method tended to produce estimates that were greater than 1.0. Although this is not recorded in the table, at worst the method produced 13 such estimates in 1000 computations.

The second modification consisted simply of computing Galton's correlation solely on the basis of the retained scores, as was done with the Pearson Extreme method. These results are given in the section headed Galton Extreme and are essentially the same as those found for the Pearson r in the same situation, on the same data. The variances for the Galton estimates, however, are generally larger than those of the Pearson method, with the differences in the two methods increasing with the population correlation.

The final modification we examined was motivated by the idea that one of the variables might in fact be relatively well researched and more information might be available for it. For instance, a person might conduct an initial survey of a group of people, compute summary statistics on the scores, and then follow up by measuring those individuals who scored in the extremes of the distribution of the screening variable. These subjects would be measured on other variables whose correlations with the screening variable are of interest. Thus this final method, called Galton Semi-extreme, used all the data for one variable (the conditionlizing variable) but only the retained data for the other.

Results of the simulation are given in Table 2. In this case the variances of the estimates are comparable to those of the Galton z method, but there is a definite negative bias apparent in the means. While one might feel that underestimation is less serious than overestimation in computing correlations, it must be remembered that this method applies only to a special and perhaps unrealistic set of circumstances.

We intend to try other modifications. Especially appealing are those of the type due to Brogden and shown in equations (3) and (5), since they do not make use of the standard deviations at all. Unfortunately they do require complete knowledge of the Y variable and two sortings of the data. More promising would be methods in which some other method of estimating the standard deviations is used, e.g., computing estimates separately in each end of the distribution and averaging, using estimates based on the range, or using the relationship between the population standard deviation and that of the tail of the conditionalizing distribution.

In closing, the relative simplicity of Galton's method makes it appealing as a tutorial device. It could be used to introduce the concepts of correlation and regression, after which Pearson's method could be introduced as the optimal computational procedure. Second, it may be the case that in some analytic situations, Galton's method may actually be preferable over the more customary representations. And, as the simulations indicate, Galton's method may be superior t Pearson's in the case of extreme group data.

Galton's contributions to statistics are often slighted in our texts. He is usually credited with introducing the concepts of correlation and regression and the symbol r. What is overlooked is the fact that he also produced a computational method for assessing correlation and regression, however non-optimal it might be. It is ironic that this method developed out of Galton's interest in the mechanics of heredity, flourished for a time, and then was driven into extinction by Pearson's method and its obviously superior adaptability. Perhaps by studying the fossilized remains of this method, we may take a page from Galton's interest in eugenics and produce a still better breed of estimators.

Let $\begin{pmatrix} X \\ Y \end{pmatrix} \sim N_2\ (\underline{\mu},\ \Sigma\ )$ with $\begin{pmatrix} X^* \\ Y^* \end{pmatrix} \sim N_2(\ \underline{0},\ R\ )$ the distribution of standard scores.

Then for any value $x_o \neq 0$ of X^*, we have

$$\frac{E(\ Y^* |\ X^* = x_o)}{x_o} = \frac{E(\ \rho_{XY}\ X^* |\ X^* = x_o)}{x_o} = \frac{\rho_{XY}\ x_o}{x_o} = \rho_{XY} \qquad (1)$$

Generalizing (Brogden, 1946),

$$\frac{E(\ Y^* |\ X^* \geq x_o)}{E(\ X^* |\ X^* \geq x_o)} = \frac{\rho_{XY}\ E(\ X^* |\ X^* \geq x_o)}{E(\ X^* |\ X^* \geq x_o)} = \rho_{XY} \qquad (2)$$

For a sample of n pairs of scores (X_i, Y_i), let p be the proportion of scores to be taken from the upper portion of the sample, and let q be the proportion of scores to be taken from the lower portion of the sample.

Let $\overline{Y}^{+}$ be the mean Y for the np largest X scores, $\overline{Y}^{-}$ for the nq smallest X scores,
$\overline{Y}_{max}$ the mean of the np largest Y scores, $\overline{Y}_{min}$ that of the nq smallest Y scores,
$\overline{X}^{+}$ the mean of the np largest X scores, $\overline{X}^{-}$ that of the nq smallest X scores,
$\overline{X}$, $\overline{Y}$ the unconditional sample means; s_X, s_Y the sample standard deviations.

Besides the Pearson coefficient, the following may be used to estimate ρ_{XY}:

Brogden (1946)
$$\frac{\overline{Y}^{+} - \overline{Y}}{\overline{Y}_{max} - \overline{Y}} \qquad (3)$$

Brown (Mosteller, 1946)
$$\frac{\overline{Y}^{+} - \overline{Y}^{-}}{\overline{X}^{+} - \overline{X}^{-}} \frac{\sigma_X}{\sigma_Y} \qquad p = \text{prop}\{X | X \geq \mu + k\sigma\},\ q = \text{prop}\{X | X \leq \mu - k\sigma\} \qquad (4)$$

Brogden-Clemans (Lord & Novick, 1968)
$$\frac{\overline{Y}^{+} - \overline{Y}^{-}}{\overline{Y}_{max} - \overline{Y}_{min}} \qquad p + q = 1 \qquad (5)$$

Galton z
$$\frac{1}{2}\left(\frac{(\overline{Y}^{+} - \overline{Y})/s_Y}{(\overline{X}^{+} - \overline{X})/s_X} + \frac{(\overline{Y}^{-} - \overline{Y})/s_Y}{(\overline{X}^{-} - \overline{X})/s_X} \right) \qquad p = q \qquad (6)$$

For Pearson's r :

$$E(\ r\ ) = \rho\ \{\ 1 - (1 - \rho^2)/2n\ \} \qquad \text{(Kendall \& Stuart, 1973)}$$

$$\text{var}(\ r\ ) = \frac{1 - \rho^2}{\sqrt{n\ (1 + \rho^2)}}\left[1 + \frac{4}{3}\frac{\rho(\rho^2 + 3)}{\sqrt{n\ (1 + \rho^2)^3}} \right] \qquad \text{(Pearson, 1896)}$$

Table 1. Means and Variances of Correlation Estimates[a]

Method	Population Correlation 0.0 Mean	0.0 Variance	0.2 Mean	0.2 Variance	0.5 Mean	0.5 Variance	0.8 Mean	0.8 Variance
Pearson[b]	-.0030	.0125	.1958	.0115	.4953	.0072	.7971	.0017
	.0	.0125	.1988	.0133	.4977	.0077	.7982	.0014
Galton z								
.1	.0020	.0210	.1998	.0196	.4991	.0132	.7970	.0046
.2	-.0035	.0161	.1982	.0154	.4943	.0101	.7955	.0032
.25	-.0038	.0156	.1937	.0143	.4933	.0095	.7954	.0031
.3	-.0054	.0157	.1907	.0144	.4928	.0098	.7957	.0030
.35	-.0058	.0161	.1911	.0154	.4917	.0101	.7952	.0033
.4	-.0079	.0171	.1930	.0163	.4904	.0108	.7959	.0035
.5	-.0099	.0189	.1901	.0187	.4909	.0134	.7974	.0044
Pearson Extreme								
.1	.0036	.0662	.3484	.0498	.7277	.0123	.9280	.0010
.2	-.0018	.0322	.2878	.0269	.6484	.0099	.8923	.0011
.25	-.0020	.0255	.2645	.0217	.6170	.0094	.8758	.0012
.3	-.0027	.0214	.2459	.0185	.5890	.0089	.8502	.0014
.35	-.0027	.0180	.2305	.0161	.5618	.0083	.8439	.0015
.4	-.0030	.0158	.2178	.0142	.5371	.0079	.8279	.0015
.5	-.0030	.0125	.1958	.0115	.4953	.0072	.7971	.0017
Pearson Same n[c]								
.1 (16)	.0	.0625	.1940	.0786	.4883	.0542	.7910	.0108
.2 (32)	.0	.0312	.1970	.0357	.4941	.0228	.7955	.0043
.25 (40)	.0	.0250	.1976	.0278	.4953	.0174	.7984	.0033
.3 (48)	.0	.0208	.1980	.0228	.4961	.0140	.7970	.0028
.35 (56)	.0	.0179	.1983	.0192	.4967	.0117	.7974	.0022
.4 (64)	.0	.0156	.1985	.0166	.4971	.0100	.7978	.0019
.5 (80)	.0	.0125	.1988	.0133	.4977	.0077	.7982	.0014

[a] Values are based on 1000 samples of size 80 unless otherwise noted.

[b] Values in the first row are empirical; those in the second row were derived analytically.

[c] Sample sizes are given in parentheses; all values were derived analytically.

Table 2

Means and Variances of Modified Methods[a]

	Population Correlation							
	0.0		0.2		0.5		0.8	
Method	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance
Galton Raw								
.1	.0009	.0217	.2007	.0204	.5012	.0160	.7986	.0077
.2	-.0039	.0167	.1985	.0190	.4981	.0122	.7971	.0059
.25	-.0046	.0163	.1943	.0151	.4954	.0118	.7973	.0060
.3	-.0095	.0165	.1912	.0154	.4944	.0119	.7975	.0058
.35	-.0067	.0170	.1914	.0168	.4934	.0124	.7973	.0062
.4	-.0087	.0182	.1932	.0175	.4922	.0133	.7980	.0064
.5	-.0110	.0207	.1907	.0209	.4930	.0165	.7989	.0078
Galton Extreme								
.1	.0035	.0693	.3481	.0513	.7271	.0140	.9289	.0014
.2	-.0050	.0354	.2862	.0303	.6452	.0124	.8907	.0019
.25	-.0052	.0292	.2585	.0252	.6135	.0120	.8742	.0024
.3	-.0066	.0258	.2393	.0225	.5851	.0120	.8590	.0028
.35	-.0067	.0229	.2248	.0212	.5570	.0119	.8425	.0030
.4	-.0085	.0214	.2142	.0200	.5318	.0120	.8270	.0032
.5	-.0099	.0189	.1901	.0187	.4909	.0134	.7974	.0044
Galton Semi-extreme[b]								
.1	.0018	.0210	.1906	.0159	.3983	.0042	.5081	.0007
.2	-.0034	.0162	.1933	.0139	.4356	.0057	.6013	.0010
.25	-.0038	.0157	.1901	.0135	.4492	.0065	.6401	.0013
.3	-.0052	.0159	.1886	.0139	.4609	.0075	.6736	.0018
.35	-.0056	.0162	.1883	.0150	.4691	.0084	.7095	.0022
.4	-.0077	.0171	.1919	.0160	.4784	.0097	.7408	.0026
.5	-.0099	.0189	.1901	.0187	.4909	.0134	.7974	.0044

[a] Values are based on 1000 samples of size 80.

[b] Standard deviation for the conditionalizing variable is based on the entire sample of 80 scores; the standard deviation for the conditional variable is based on only the extreme groups.

References

Brogden, H. E. On the interpretation of the correlation coefficient as a measure of predictive efficiency. Journal of Educational Psychology, 1946, 37, 65-76.

Galton, F. Typical laws of heredity. Proceedings of the Royal Institute, 1877, 8, 282-301.

Galton, F. Regression towards mediocrity in hereditary stature. Journal of the Royal Anthropological Institute, 1885, 246-258.

Galton, F. Co-relations and their measurement, chiefly from anthropometric data. Proceedings of the Royal Society, 1888, 45, 135-145.

Galton, F. Memories of my life. New York: E. P. Dutton, 1909.

Kendall, M. G., & Stuart, A. The advanced theory of statistics (3rd ed., v. 2). London: Charles Griffin, 1973.

Lord, F. M., & Novick, M. R. Statistical theories of mental test scores. Reading, Mass.: Addison-Wesley, 1968.

Mosteller, F. On some useful "inefficient" statistics. Annals of Mathematical Statistics, 1946, 17, 377-408.

Pearson, K. Mathematical contributions to the theory of evolution - III. Regression, heredity, and panmixia. Philosophical Transactions of the Royal Society of London, 1896, A-187, 253-318.

Weldon, W. F. R. Certain correlated variations in Crangon vulgaris. Proceedings of the Royal Society, 1892, 51, 2-11.

Recommended Sources

Forrest, D. W. Francis Galton, the life and work of a Victorian genius. New York: Taplinger, 1964.

Walker, H. M. Studies in the history of statistical method. New York: Arno Press, 1975. (Originally published, 1929.)